Analysis of Tooth Interior Fatigue Fracture Using Boundary Conditions from an Efficient and Accurate Loaded Tooth Contact Analysis

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Abstract
This paper demonstrates a modification to the analysis of Tooth Interior Fatigue Fracture, as implemented in SMT’s MASTA software, in which Loaded Tooth Contact Analysis (LTCA) results from a specialised 3D elastic contact model have been utilized to determine the load boundary conditions for analysis of Tooth Interior Fatigue Fracture (TIFF) to the method of MackAldener. In contrast to MackAldener’s method, using finite element contact analysis, this method allows for quick analysis times for the calculation of the stress history leading to fast optimization. The method considers the effect of case hardening by applying a constant volume expansion to an FE model of the tooth and the resulting residual stresses are superimposed to stress history results. Once the series of stress history steps have been obtained, Findley multiaxial fatigue criteria are used to determine the risk of fatigue crack initiation. This paper reproduces results obtained by MackAldener using the proposed methodology and good agreement is observed.

1 Introduction
Gears are case hardened to produce residual compressive stresses at the surface which improve wear resistance, bending and contact fatigue strength. These compressive stresses are balanced by tensile stresses in the core. This poses an increased risk of fatigue crack initiation in the material below the surface. The failure mode where a subsurface fatigue crack initiates close to case-core boundary, approximately mid-height of the tooth, is called Tooth Interior Fatigue Fracture (TIFF) or Tooth Flank Fracture (TFF, sometimes also known as Tooth Flank Breakage or TFB). The location of the crack initiation distinguishes this failure mode from other fatigue failure modes. Previous research [1-8] has established that the directions in which the crack progresses and the appearance of the associated fracture surface is dependent on the flank loading. Although there does not appear to be total agreement in the literature TIFF (failure with reverse loading) and TFF (failure with single flank loading) appear to have very similar characteristics and crack initiation mechanisms. The final fracture shape is different, due to TIFF having near symmetric total stresses along the tooth centreline (with two possible initiation points per tooth). They can be analysed using the same approaches.

This type of failure appears at loads below the allowable loading conditions for pitting and bending fatigue failure modes based on standard calculation procedures [9]. Therefore understanding of such failure is required at the design stage of transmission systems.
In this section we give a summary of the current calculation methods found in the literature for both TIFF and TFF. The currently proposed approaches for TIFF and TFF all have very similar fundamental approaches consisting of 4 stages:

- Calculation of stress history
- Calculation or specification of residual stresses
- Calculation of equivalent stresses using some fatigue criteria
- Comparison with some initiation thresholds

The differences between the methods lie in the details of the above steps. Further the applicability of the methods depends on the implementation details of these steps and the assumptions made at each stage.

The calculations of each step could be interchanged between methods creating a number of permutations of possibilities. TIFF and TFF has been shown to depend on gear macro geometry, loading, material and hardening properties. Currently there is no accepted international standard to assess the probability of TIFF and TFF failures for a given design and therefore no standardised method to assess the relative importance of the influencing factors.

**Tooth Interior Fatigue Fracture Calculation Methods**

MackAldener [1-3] has shown that an analysis method based on two-dimensional Finite Element Analysis (FEA) can be utilized to analyse the risk of TIFF and determine the optimum macro geometry, material and case hardening properties. In this analysis MackAldener used the gear Loaded Tooth Contact Analysis program LDP to calculate the total force on one tooth at different phases within the mesh cycle. The calculated force was then applied to the FE mesh as a torque after normalising with the face width. A contact analysis was then run on the 2D FE model in order to calculate the stress history. MackAldener’s early paper [1] assumed constant material properties for case and core. However because the gear is case hardened, the materials fatigue properties will not be the same throughout the tooth. This assumption was removed in later papers where non-homogenous fatigue properties were used in the case region. Fatigue properties were assumed to vary with depth in the same way as the hardness profile. The methodology used in this paper is based on MackAldener’s FE based analysis method and therefore further details of MackAldener’s methodology and results are discussed in later sections.

Due to the complexity of setting up and running MackAldener’s FE based method within a general FE package, MackAldener [2] also proposed a simpler semi analytical method for rapid calculation, design parameter studies and optimisation, but with some compromise in the accuracy of results. In the analysis results presented in [2], results differed between this method and MackAldener’s FE based method by a maximum of 20%. This method was seen to over predict the risk of TIFF.

MackAldener concluded that TIFF can be avoided if the slenderness ratio is reduced, tensile residual stresses are reduced, the gear is not used as an idler gear and optimum case and core properties are used. MackAldener’s TIFF methodology can be used to investigate TFF as well as TIFF, simply by considering single flank loading instead of reverse loading. However crack initiation thresholds could be different and require experimentation or field data to clarify.

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1 The ISO committee is currently working on a draft standard, ISO/DTR 19042, for the calculation of Tooth Flank Fracture performance.
Tooth Flank Fracture Load Capacity Calculation Methods

To the author’s knowledge there are two main TFF load capacity calculation methods proposed in the literature.

The first is a calculation method for TFF load capacity proposed by Witzig [8]. This method has been published in Witzig [8], Tobie et al. [6] and Boiadjev et al. [7]. The method relies on calculation of the local stress history based on a shear stress intensity hypothesis of Foppl [10]. The method has significant empirical contributions and is limited in applicability due to the empirical nature of the equation used in the shear stress intensity hypothesis. In the literature this equation has been presented for single contact point only, it could in theory be extended to consider two contact points (i.e. reverse loading) but this is not trivial. The method as published is also restricted to case hardened gears due to the assumptions in the residual stress calculation. It should also be noted that the proposed method in its current form can under estimate the crack initiation risk if resulting tensile stresses within the core are not small, since these resulting tensile stresses are currently neglected. However, this simplification may only be reliable when the core section is much larger when compared to the thickness of the case. This introduces limitations on applicability for slender teeth and extensive case hardening depths.

Ghribi and Octrue [5] proposed an alternative calculation method for TFF load capacity. This method is more generic than that of Witzig [8] and can be applied to both Tooth Flank Fracture and TIFF. The method proposes use of a multiaxial fatigue criterion and considers the importance of including tensile stresses in the core. The stress history is calculated using the Hertzian contact calculations of ISO TR 15144-1 [11], together with a proposal of Johnson for the stress at a depth inside the tooth. Method A of ISO TR 15144-1 [11] is based on using the results of a 3D gear Loaded Tooth Contact Analysis, however only contact stresses calculated by the standard have been considered in these analyses. The addition of stresses due to bending has been mentioned as planned future work.

None of these methods include an FEA based methodology. Although they clearly could be adapted to do so. However, using general FE packages, this requires considerable time and computational power to set-up and run analyses.

This section has given a brief literature review and discussed the motivations for this work. In the following section we outline the models used within this paper. Justification is given for model selection and the formulation described is used throughout the remainder of this paper. Section 3 demonstrates the use of the described methodology as a means to investigate the risk of fatigue crack growth beneath the surface. The influence of various parameters has been investigated. Good validation against previously published data is shown for the prediction of Crack Initiation Risk Factor. The limitations and potential applications of the modelling strategy are then discussed. Finally, Section 4 presents the main conclusions and identifies recommendations for future work.

2 Methodology / Analysis

This section outlines the methodology used for the analysis of TIFF. The presented methodology has been implemented in SMT’s MASTA Release 7.

MASTA’s implementation is derived from MackAlderner’s finite element method but the need for a full FE tooth contact analysis has been removed by using loading conditions calculated using MASTA’s specialised Loaded Tooth Contact Analysis. MackAldener also simplified his FE analysis in a later stage, not for the calculation of Crack Initiation Risk Factor but when investigating the crack propagation
mechanism during the TIFF [12]. This method removes the complexity of the contact analysis and speeds up the calculation while reducing the computational requirements.

**Analysis of the Stress History**

MASTA’s 3D Loaded Tooth Contact Analysis model combines an FE representation of bending and base rotation stiffness of the gear teeth and blank with a Hertzian contact formalism for the local contact stiffness. This calculation includes the effect of extended tip contact where the effective contact ratio is increased under load due to tooth bending. This effect can be particularly important for slender tooth gears which are also more at risk of TIFF.

This model is used to determine load boundary conditions at a selected number of time steps through the mesh cycle. At each time step the load distribution between and across the teeth is calculated and at each of the contact lines the following are obtained:

- Load positions
- Load magnitudes
- Hertzian half widths

A separate fine 2D mesh of the gear tooth is then built automatically using plane strain elements. At each time step within the mesh cycle the position and the distribution of the load is determined from the results of the 3D tooth contact analysis and applied to the 2D FE mesh using the average load position and Hertzian half width. The force is assumed to lie in the line of action and is normalized by the tooth width (definition used by MackAldener). The contact force distributed onto the 2D FE model assuming a Hertzian distribution within the Hertzian contact width calculated using the 3D LTCA model. In the results presented below the finite element mesh was sized according to the Hertzian half width and a refinement study was performed to check the convergence of the results.

**Residual stress analysis**

Residual stresses influence the stress states within the gear tooth. These stresses are not load dependent and assumed to be constant over time. Residual stresses due to case hardening and shot peening are superimposed. A separate FEA analysis using the 2D mesh used to calculate the stress history due to flank loading is used to calculate these residual stresses.

The transformation strain profile is isotropic and measured relative to the core. This profile has been presented as a piecewise polynomial with smooth connections by MackAldener [3].

\[
\varepsilon_t(z) = \begin{cases} 
\varepsilon_1 + 4(\varepsilon_2 - \varepsilon_1) \left( \frac{z}{\bar{z}} - \left( \frac{z}{\bar{z}} \right)^2 \right) & \text{if } 0 \leq z \leq \frac{\bar{z}}{2} \\
-4\varepsilon_2 \left( 1 - 6 \left( \frac{z}{\bar{z}} \right) + 9 \left( \frac{z}{\bar{z}} \right)^2 - 4 \left( \frac{z}{\bar{z}} \right)^3 \right) & \text{if } \frac{\bar{z}}{2} \leq z \leq \bar{z} \\
0 & \text{if } \bar{z} \leq z
\end{cases}
\]

where, \( \varepsilon_1 \) is the transformation strain at the surface, \( \varepsilon_2 \) is the maximum transformation strain, \( z \) is the normal depth at the point considered and \( \bar{z} \) is the total case depth.

The volume expansion in the surface layer due to the case-hardening process is modelled by applying a temperature profile. The temperature profile applied is the same as the transformation strain profile.
when the coefficient of thermal expansion is set to 1. All side nodes are allowed to move only in the radial direction.

Figure 1 shows the results presented by MackAldener for the variation of residual stresses with depth beneath the surface both using the analysis method above and from measurements carried out by MackAldener. This residual stress profile is a result of the transformation strain profile. Figure 1 further compares this residual stress profile with that proposed by Lang [13] and used by Witzig [8] in the investigation of TFF. Interestingly the profiles differ quite significantly. This may be due to a significant material dependency not considered, but the reason is not currently known and needs to be understood further.

![Figure 1: Variation of residual stresses with increasing depth. The total case depth is marked by a dashed line.](image)

In this study the residual stress profile which is the result of the strain profile has been utilized with $\varepsilon_1 = 0.000833$ and $\varepsilon_2 = 0.00114$ as determined by MackAldener [3]. It should be noted that the resulting calculated residual stresses can change from one mesh position to another due to the variation in tooth thickness.

**Final Stress State and Fatigue Crack Initiation Criterion**

The effective stress state within the gear teeth during its load cycle is calculated, without calculating residual stresses at each step, by superimposing the calculated stress history states and the estimated residual stresses.

The Findley multiaxial fatigue criterion [14] is then used to analyse the stress history and assess if failure is going to occur. Within our analysis the Findley critical plane stress has been calculated for every 5 degrees of inclination at each node. The value of 5 degrees was chosen, instead of every 1 degree used by MackAldener [2], as results did not show a significant dependency on this value. This was confirmed by the cases shown in the initial of the results section of this paper where the difference between using an inclination increment of 2.5 degrees is less than 0.05%.

The Findley stress is calculated as:

$$\sigma_F = \tau_a + a_{cp} \sigma_{n,max}$$
where \( \tau_a \) is the shear stress amplitude, \( \sigma_{n,\text{max}} \) is the maximum normal stress and \( a_{cp} \) is the material fatigue sensitivity to normal stress. Variation of the material properties within the tooth are related to the hardness profile as described below.

The ratio between the maximum Findley critical plane stress and critical shear stress, is a measure of the risk of crack initiation. This metric is called the Crack Initiation Risk Factor, CIRF.

**Hardness profile and material properties**

The variation of the material properties within the case and core play an important role in TIFF. However, many assumptions have been made in previous analyses in this area. Because the analysed gear is case hardened the material properties are not the same throughout the tooth. Hence, the critical shear stress, and the fatigue sensitivity to normal stress, in the critical plane criterion are also expected to vary from place to place. As with MackAldener, for our analysis we have assumed that these properties vary in the same way as an assumed hardness profile.

\[
H(z) = H_{\text{surface}} g \left( \frac{z}{\bar{z}} \right) + H_{\text{core}} \left( 1 - g \left( \frac{z}{\bar{z}} \right) \right)
\]

\[
g \left( \frac{z}{\bar{z}} \right) = 1 - 3 \left( \frac{z}{\bar{z}} \right)^2 + 2 \left( \frac{z}{\bar{z}} \right)^3
\]

where, \( H_{\text{surface}} \) is the hardness at the surface, \( H_{\text{core}} \) is the hardness at the core, \( g \) is the function which determines the variation between the case and the core defined by MackAldener, \( z \) is the normal depth at the point considered and \( \bar{z} \) is the total case depth.

Figure 2 shows a comparison of the hardness profile measurement and curve fit proposed by MackAldener with other hardness profile models in the literature. For this article MackAldener’s curve fit has been used. It is interesting to note that the hardness profile model proposed by Thomas [15] has been found to give the best comparison against MackAldener’s measurement that of Tobe et al. [16] is also close. The method of Lang could lead to a difference in the CIRF since fatigue properties are expected to differ near the case core boundary.

![Figure 2: Experimentally measured hardness profile and curve fits of MackAldener together with a number of empirical models available in the literature. The total case depth of 1.2 mm is marked by a dashed line. Effective case depth where hardness drops below 550HV is 0.68 mm (required for empirical models).](image)
In the current implementation, the material properties are assumed to vary continuously between case and core in the same manner as the hardness profile. This assumption is not required if the variation of the material properties are known.

Critical shear stress, $\sigma_{\text{crit}}$

$$\sigma_{\text{crit}}(z) = \sigma_{\text{crit, surface}} g\left(\frac{z}{Z}\right) + \sigma_{\text{crit, core}} \left(1 - g\left(\frac{z}{Z}\right)\right)$$

Fatigue sensitivity to normal stress, $a_{cp}$

$$a_{cp}(z) = a_{cp, \text{ surface}} g\left(\frac{z}{Z}\right) + a_{cp, \text{ core}} \left(1 - g\left(\frac{z}{Z}\right)\right)$$

The surface fatigue resistance of a gear flank and root can be improved by shot-peening. This improvement is due to an increase in the compressive stresses in a thin layer close to the surface. This layer is very thin compared to the case hardening layer. Shot peening properties for depth and the effect on the critical shear stresses are required. In the current implementation the depth has been assumed constant and the effect on the critical shear stresses assumed to be constant, specified via shot peening factor.

**Assumptions**

There are a few points worth noting about the assumptions used within our methodology:

- Two-dimensional analysis has been utilized to analyse risk of TIFF. This option has been considered to be enough by MackAldener [1].
- Material fatigue properties vary in the same way as the hardness profile. These material properties within the case/core boundary an important role in TIFF.
- The transformation strain profile is isotropic and measured relative to the core. It is also assumed that this profile could be represented by a piecewise polynomial with smooth connections.
- It is assumed that the residual stresses can be superimposed stress history.
- No micro geometry has been included in the FE mesh, but loading conditions include selected micro geometry.
- Friction is not currently included in the LTCA analysis which provides the load boundary conditions to the TIFF analysis.

### 3 Validation Results

This section is split into 2 parts: The first compares contour plots for a single design stage to results obtained by MackAldener, the second replicates the factorial design experiment run by MackAldener with the results for all 32 designs compared against the reported CIRF values in the literature.

Firstly contour plots of the calculated CIRF for a single design are compared with MackAldener’s results. Figure 3 shows the results for both single stage and idler loading of the gear. For comparison, previously proposed [1, 4] crack propagation paths are also shown in Figure 3. In general the correlation of results is good. It is clear from Figure 1 that MASTA’s results are smoother compared to those of MackAldener, as many more time steps have been considered within the mesh cycle in the MASTA solution. Table 1 shows the material factors used within this analysis, with the nominal values
used for the results of Figure 3. Other material properties, considered constant throughout the entire study, are critical shear stress, 1090 MPa, and sensitivity to normal stress within the case, 1.

<table>
<thead>
<tr>
<th>Single stage usage</th>
<th>Idler usage</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Diagram" /></td>
<td><img src="image2" alt="Diagram" /></td>
</tr>
<tr>
<td><img src="image3" alt="Diagram" /></td>
<td><img src="image4" alt="Diagram" /></td>
</tr>
</tbody>
</table>

Figure 3: Comparison of CIRF for single stage and idler loading as calculated by MackAldener and MASTA. Results are presented for the original gear with nominal factors as described in Table 1 and Table 2. Expected crack propagation paths for each loading type are shown.
Factorial Design

MackAldener conducted a factorial design with five factors. In total 32 designs have been considered by varying critical plane stress within the core (A), fatigue sensitivity to normal stress within the core (B), gear tooth geometry (C), total case depth (D) and torque on the pinion (E). For each of the factors two levels, low and high, have been considered, their values are presented in Table 1. Details of the gear tooth geometries are given in Table 2. For each of the designs the CIRF throughout the tooth was calculated.

Table 1: Summary of factors used in the factorial design [2]

<table>
<thead>
<tr>
<th>Factor</th>
<th>Description</th>
<th>Low level</th>
<th>Nominal level</th>
<th>High level</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Critical plane stress within the core</td>
<td>359.85 MPa</td>
<td>479.8 MPa</td>
<td>559.75 MPa</td>
</tr>
<tr>
<td>B</td>
<td>Fatigue sensitivity to normal stress within the core</td>
<td>0.28</td>
<td>0.37</td>
<td>0.46</td>
</tr>
<tr>
<td>C</td>
<td>Gear design</td>
<td>Slender</td>
<td>Original</td>
<td>Not-Slender</td>
</tr>
<tr>
<td>D</td>
<td>Total case depth</td>
<td>0.9 mm</td>
<td>1.2 mm</td>
<td>1.5 mm</td>
</tr>
<tr>
<td>E</td>
<td>Torque on the pinion</td>
<td>1238 Nm</td>
<td>1651 Nm</td>
<td>2064 Nm</td>
</tr>
</tbody>
</table>

Table 2: Gear data for the gear designs considered [2]

<table>
<thead>
<tr>
<th></th>
<th>Slender</th>
<th>Original</th>
<th>Not-slinger</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pinion</td>
<td>Wheel</td>
<td>Pinion</td>
<td>Wheel</td>
</tr>
<tr>
<td>Modulus [mm]</td>
<td>2.34</td>
<td>3.06</td>
<td>3.75</td>
</tr>
<tr>
<td>Pressure angle [°]</td>
<td>17.5</td>
<td>20</td>
<td>22.5</td>
</tr>
<tr>
<td>Helix angle [°]</td>
<td>15</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Centre distance [mm]</td>
<td></td>
<td>166.5</td>
<td></td>
</tr>
<tr>
<td>Face width [mm]</td>
<td>43</td>
<td>35</td>
<td>43</td>
</tr>
<tr>
<td>Tip diameter [mm]</td>
<td>116.5</td>
<td>230.6</td>
<td>116.5</td>
</tr>
<tr>
<td>Profile shift coefficient</td>
<td>0.270</td>
<td>-0.669</td>
<td>0.65</td>
</tr>
<tr>
<td>Protuberance [mm]</td>
<td>0.06</td>
<td>0.095</td>
<td>0.06</td>
</tr>
<tr>
<td>Addendum for tool [mm]</td>
<td>4.85</td>
<td>3.9</td>
<td>4.35</td>
</tr>
<tr>
<td>Protuberance angle [°]</td>
<td>5.044</td>
<td>8.342</td>
<td>3.045</td>
</tr>
<tr>
<td>Tool edge radius [mm]</td>
<td>0.45</td>
<td>0.85</td>
<td>1.2</td>
</tr>
</tbody>
</table>

Figure 4 shows a comparison of the calculated maximum CIRF for all 32 designs. From Figure 4 it is clear that there is a good overall correlation between CIRF calculated in MASTA and by MackAldener [2]. Some slight deviation in results in seen for designs 13 through 16. These experiments relate to varying torques and varying total case depth for low critical shear stress and high fatigue sensitivity to normal stress within the core for not-slender gear set.
Figure 4: Comparison of the calculated Maximum CIRFs from MASTA and MackAldener’s finite element calculations. Index values have been determined by first sorting according to factor (A through E) then sorting the values of each factor in ascending order.

Figure 5 displays the average CIRF results for each value of each factor together with the average for some interactions. It can be seen that good agreement has been observed for factors A, B, D and E and reasonable agreement has been observed for factor C. As discussed above, the lower level of agreement shown for factor C is due to deviations in experiment index 13 to 15. These experiments relate to varying torque and varying total case depth for low critical shear stress and high fatigue sensitivity to normal stress within the core for not-slim gear set. Interaction effects which include this factor also affected by this deviation.

Figure 5: Comparison of main and interaction effects on CIRF response for MASTA and MackAldener’s FE calculations. The dotted line represents the mean CIRF response.

The trends shown in Figure 5 can be used to propose a strategy to avoid a high CIRF. Crack initiation risk can be mitigated by using a material with higher critical shear stress and lower fatigue sensitivity to normal stress within the core, increasing the total case depth and reducing gear tooth slenderness and transmitted torque.
Conclusions

An analysis technique based on finite element analysis to analyse risk of TIFF crack initiation has been presented. In this analysis Loaded Tooth Contact Analysis (LTCA) results obtained a full 3D elastic tooth contact model have been utilized to determine load boundary conditions.

The key methodologies and conclusions from this paper are as follows:

- It is possible to replace a computationally expensive explicitly modelled FE based contact analysis with simple load boundary conditions obtained by a separate specialised gear Loaded Tooth Contact Analysis, in order to apply MackAldener's methodology for the analysis of TIFF.
- Smoother results have been obtained compared to those of MackAldener's results, due to the number of time steps considered within the mesh cycle.
- A parametric study initially conducted by MackAldener to investigate which parameters influence the risk of TIFF has been repeated to validate the proposed methodology and implementation and good agreement was achieved.
- MackAldener's findings with regards to possible design options for avoiding TIFF have been confirmed in this study.

Further work on gears loaded on a single flank (i.e. TFF) and comparison of TIFF load carrying capacity with that for other failure modes, such as bending and pitting fatigue, are planned for future work.

References