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Asymmetric Cylindrical Gears

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ABSTRACT: Cylindrical gears with asymmetric pressure angles and fillet geometry are now of interest to the automotive industry. Benefits of asymmetric gears are typically seen for gears which operate mainly on a single flank. Higher pressure angles on the active flanks and lower on the inactive, can lead to designs with high strength, low NVH, reduced weight and improved lubricant film thickness. Standards do not cover the design and rating of asymmetric gears. Loaded Tooth Contact Analysis provides an efficient way to assess their benefits for durability, NVH and efficiency. We present a Loaded Tooth Contact Analysis model and explore the application of asymmetric gears within an automotive transmission.

KEY WORDS: **Standardised, Vibration, noise and ride comfort, gear noise, CAE.** Free, Asymmetric Gears, Loaded Tooth Contact Analysis (B3)

1. INTRODUCTION

In typical applications the two flanks of a given cylindrical gear have different operating conditions with, for example, different loads and different periods of operation. This is the case for automotive gear trains where the operation is mostly unidirectional with the primary drive flanks operating for a much greater time and under greater load than the coast flanks. Asymmetric cylindrical gears using a different pressure angle on each flank can be designed to improve the performance on the drive flank at the expense of the coast. Asymmetric gears have been used for many years especially in high cost, low volume applications such as wind and aerospace. Significant application and interest for asymmetric gears is now being shown within the automotive industry. With this increasing interest comes an increasing need for methods and tools to assess the relative merits of asymmetric gears as compared to symmetric gears and assess the impact of changes in asymmetric gear geometry. The standard rating methods for symmetric cylindrical gears are not directly applicable to asymmetric gears. In this paper we present a Loaded Tooth Contact Analysis method for asymmetric gears which provides an accurate and efficient design tool for analyzing and comparing designs. The presented method is implemented in SMT's MASTA software. We further present an example comparative study using this tool for an example automotive application.

2. ASYMMETRIC GEAR GEOMETRY AND RATING

2.1. Drive Side Geometry

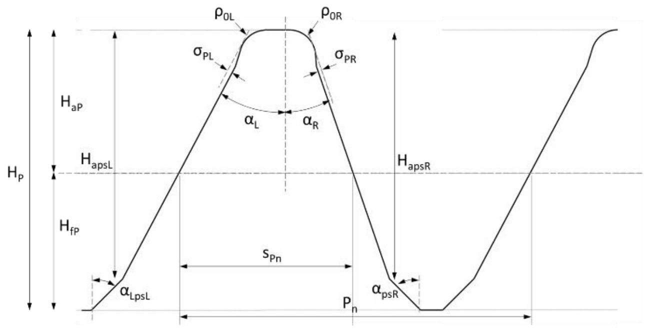
Asymmetric cylindrical gears are involute cylindrical gears with asymmetric flank profiles. In particular, the usual approach is to increase the operating pressure angle on the drive flank beyond the traditional limits of symmetric gears by using a lower pressure angle on the coast flank to maintain sufficient tip thickness. Such a design can lead to benefits including an increased transverse contact ratio on the drive side, leading to lower sliding and therefore less scuffing risk and higher efficiency. The increased pressure angle on the drive flank results in a smaller base radius which gives a higher normal load for a given torque however it also lead to a larger radius of curvature at contact, potentially leading to lower contact stresses. Decreased bending stresses can also result due to a decreased bending moment on the gear tooth. Higher strength on the drive side can lead to more compact, lower weight designs.

2.2. Coast Side Geometry

With an increased pressure angle on the drive side, a decreased pressure angle on the coast side is required to maintain tip thickness. This decreased pressure angle often leads to NVH benefits for the drive side with the increased tooth compliance. One of the biggest challenges when designing asymmetric gears for applications where operation on both flanks does occur is to limit the decrease in performance on the coast side. In an automotive application, for example, particular attention should be paid to NVH performance in coasting conditions.

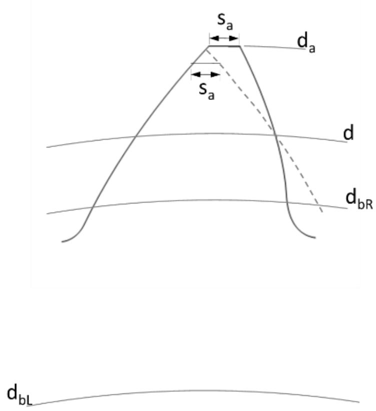
It is worth noting that for idler gears, operating on both flanks in the same operating conditions, there may be additional benefits with asymmetric gears. In a planetary system the planet gears operate on both flanks. Typically the sun to planet mesh fails before the planet to annulus. Using a higher pressure angle on the sun side and lower on the annulus, the lines between the meshes can be balanced.

Figure 1 shows the major geometry parameters for an asymmetric rack cutter gear while Figure 2 shows the geometry parameters for an asymmetric gear. There are single normal module, helix angle and tip and root diameters. However there is asymmetry in pressure angle, root geometry and chamfer geometry.



Single Value	Defined per Flank
Normal Module	Pressure Angle
Helix Angle	Edge Radius and Protuberance
Addendum and Dedendum	Chamfer Definition

Fig. 1 Asymmetric Rack Geometry



Single Value	Defined per Flank
Reference Diameter	Base Diameter
Tip and Root Diameters	Tip and Root Form Diameters
	Finish Stock

Fig. 2 Asymmetric Gear Geometry

2.2.1. Rating

The existing cylindrical gear rating standards for the major gear failure modes of e.g. pitting and bending in ISO 6336[(1)] are not directly applicable to asymmetric gears. Some authors have applied the methods of these standards with modifications[(2),(3),(4),(5)]. Kapelevich, for example, has developed a rating method which utilizes the existing standards and equivalent symmetric tooth gears, with conversion factors based on FE analysis. Kapelevich has reported good results for the method although it is not entirely satisfactory from a physical perspective as it does not directly model the actual situation. Langheinrich[(4)] on the other hand developed an approach by modifying the equations of DIN 3990/ISO 6336. Sekar and Muthuverappan[(5)] adapted the form and stress correction factors of ISO 6336 Method B for spur asymmetric gears.

In this paper we present an approach to the analysis of asymmetric gears based on a high fidelity hybrid Hertzian and FE based specialized gear Loaded Tooth Contact Analysis. This analysis method is described in the next section.

3. LOADED TOOTH CONTACT ANALYSIS

For the assessment of asymmetric gear tooth contact conditions, including load distribution, transmission error and root and contact stresses, a hybrid Hertzian and FE based Loaded Tooth Contact Analysis method was developed based on the model presented in Langlois et. al[(6)] for symmetric gears.

3.1. Hybrid Hertzian and FE based LTCA Model

The developed model is a specialized gear loaded tooth contact analysis model. The analysis is quasi static, performed at n discrete time steps. At each time step, firstly the potential contact points on the gear teeth flanks in mesh are calculated. The assumption is made that deflections of the system are sufficiently small that the potential contact points and normals do not move from their theoretical no-load locations. Applied loads can bring those points into and out of contact however do not move those points locations. These potential contact points are calculated from the cylindrical gear theoretical contact lines under no misalignment and no micro geometry. In addition to these “nominal” potential contact points a set of additional potential contact points are included at the tips of the gear teeth which are points which can potentially come into contact early, prematurely, due to deflections under load[(6)].

Compatibility and force equilibrium conditions are set up between the sets of potential contact points.

$$U_{k1}+U_{k2}+\varepsilon_k-\alpha \geq 0$$

Where:

1, 2 label the pinion and wheel respectively

U_{ki}- Is the elastic deformation of gear i at point k

ε_k – Is the initial separation at point k

α – Is the rigid body approach

$$\sum_k F_k = F$$

Where:

F_k – Is the normal force at strip k

F – Is the total applied normal force due to the applied torque

The first equation enforces that there is no penetration between the contacting points. The second enforces that the sum of calculated forces is consistent with the applied torque input.

The elastic deformations U_{ki} are a function of the forces and so these equations must be solved iteratively for α, which is related to the transmission error, and F_k. For the calculation of the elastic deformations the stiffness contributions are separated into two parts. For the bulk bending stiffness of the teeth and base rotation of the teeth on the gear body an automatically generated FE model of the gear macro geometry is used. This model is easily adaptable from symmetric to asymmetric cylindrical gears simply by using the asymmetric gear geometry for this FE model. For the contact stiffness local to the contact points the formalism of Weber[7] is used.

Once the load distribution across the flanks has been calculated the contact pressures are calculated as a post calculation with a Hertzian cylinder on cylinder formalism with the radius of curvatures given by the roll distance of the contact points. Root stresses are post calculated by applying the calculated load distribution back on to the FE model and reading the stresses in the root area of the FE model directly.

Due to this separation between the local contact stiffness and the bulk tooth bending and base rotation stiffness, the FE model required for the calculation can have a coarse mesh. The FE mesh is not being used to solve the Herztian contact, as this is solved by Weber's formalism. In contrast, to perform gear loaded tooth contact analysis in a general FE package a very fine mesh is required at the contact points in order to capture the local Hertzian contact deformations. As a result, the specialized gear contact model takes the order of seconds to run a load condition while a general FE package takes orders of magnitude longer. The method therefore leads to a viable design tool where multiple loads, design parameter changes and tolerance studies can be run within the design process.

3.2. Validation of the Model

The specialized gear Loaded Tooth Contact Analysis method for asymmetric gears described in the previous section was validated against a surface to surface contact analysis model in the general finite element software ANSYS. Code was written to set up the finite element model and analysis using the ANSYS parametric design language (APDL). The node positions in the FE model were defined directly from an analytical description of the geometry, including modifications to these positions for micro geometry modifications. No CAD model was used. Figure 3 shows a schematic of the ANSYS model set up including the applied boundary conditions. The geometry parameters for one of the examples used for validation is given in Table 1. This particular validation example is not an automotive example. It was chosen as it has an extreme asymmetric geometry with 38 and 19 degree pressure angles on drive and coast flanks respectively and was introduced by Kapalevic[2]. 15μm of lead crowning and 13μm of parabolic profile crowning was applied to the pinion. The gears are steel.

Table 1 Asymmetric gear pair validation example geometry.

	Pinion	Wheel
Number of Teeth	27	41
Face Width (mm)	30	28
Normal Module (mm)	3	
Helix Angle (°)	0	
Centre Distance (mm)	102	
Tip Diameter (mm)	87.09	128.935
Root Diameter (mm)	74.393	116.23
Cutter Edge Radius (mm)	0.75	0.75
	Drive	Coast
Pressure Angle (°)	38	19
Contact Ratio	1.2578	1.7233

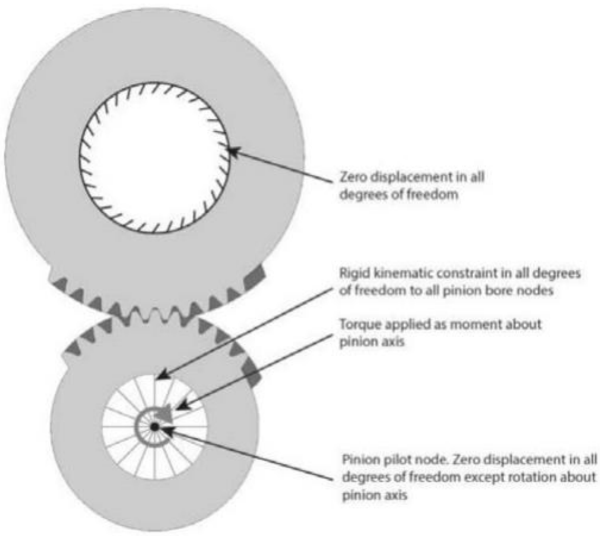


Fig. 3 Schematic diagram showing the displacement and force boundary conditions applied to the FE model

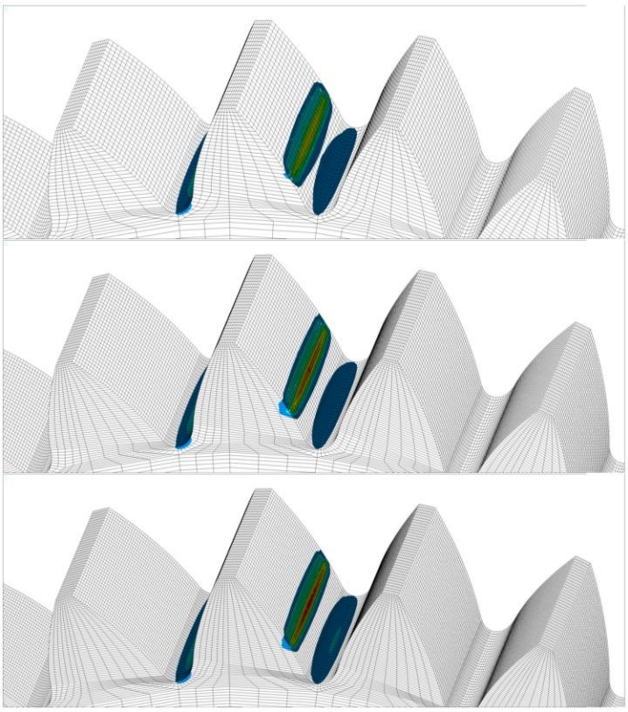


Fig. 4 ANSYS meshes; from top to bottom – Mesh 1, Mesh 2, Mesh 3

To check the accuracy of the FE model results a mesh convergence study was performed. Figure 4 shows the levels of meshes used in order to achieve convergence.

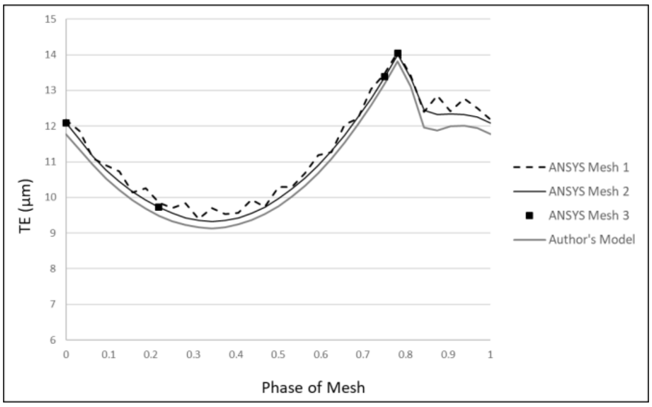


Fig. 5 ANSYS convergence study at 100 Nm Torque on drive flank. Transmission Error (µm)

Figure 5 shows the result of one such convergence study together with the corresponding results of the author's model. TE is shown for the torque value for which the results were seen to be most sensitive to the FE mesh size. In this example Mesh 1 is seen to give a good prediction of mean and peak-peak TE compared to the other meshes however the TE trace is not 100% smooth. Mesh 2 is seen to be smooth and gives almost identical results to Mesh 3. Similar results were seen at all loads considered, from 100 Nm – 1000 Nm. A similar convergence study was performed for the results of the author's specialized LTCA model. Excellent correlation is seen between the Author's model and ANSYS.

Figure 6 shows peak-peak transmission error against load and Figure 7 shows mean transmission error against load for the author's model and the full ANSYS analysis.

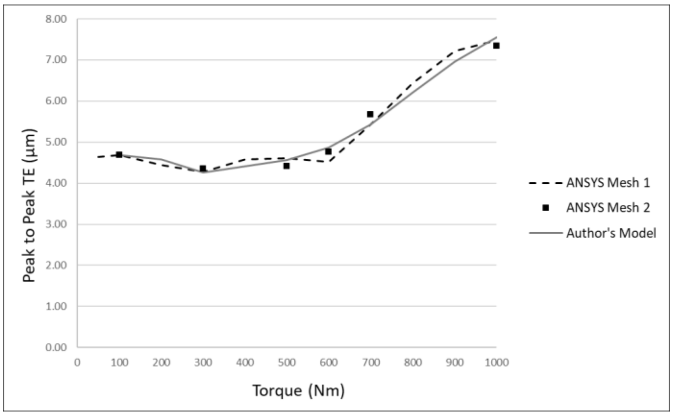


Fig. 6 Comparison of Author's model and ANSYS. Peak-Peak Transmission Error (µm) against load.

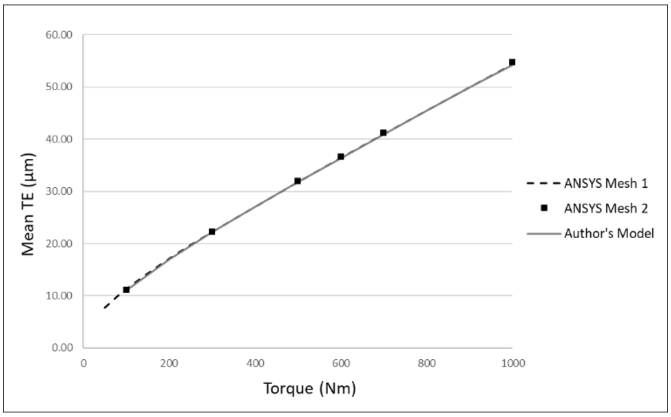


Fig. 7 Comparison of Author's model and ANSYS. Mean Transmission Error (µm) against load

Figure 8 shows the results for the maximum principal root stress, in tension, for the pinion.

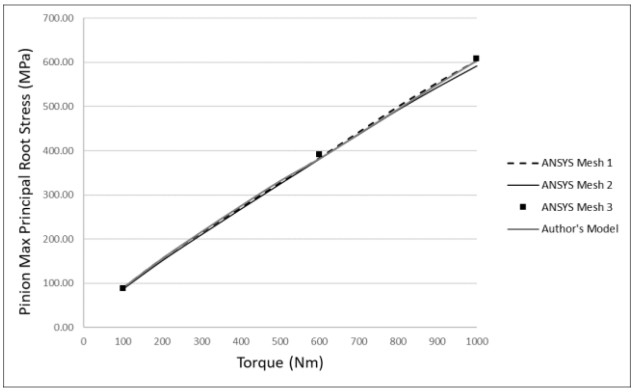


Fig. 8 Comparison of Author's model and ANSYS. Pinion Max Principal Root Stress (MPa) against load

Finally Figure 9 shows a comparison of the maximum contact pressure. The results for maximum contact stress are taken in the region away from any severe tip contact. It is very difficult to calculate an accurate value for the stress in edge contact regions such as extended tip contact, both via full FE or specialized gear contact analysis. In such regions the actual contact stress will be a function of the details of the actual tip shape in terms of manufacture and wear under operating. It is important to identify when such contact occurs, which such models can do, and include micro geometry such as tip and root relief in designs to avoid hard tip contact.

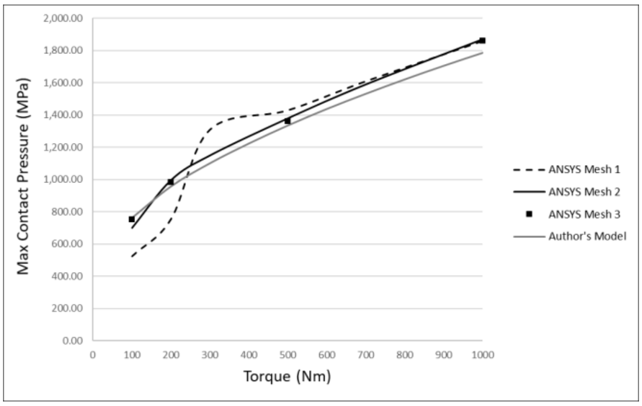


Fig. 9 Comparison of Author's model and ANSYS. Maximum Contact Stress (MPa) against load

It is worth noting that the run times for the ANSYS model on a typical desktop with 64 GB RAM, Intel Core i7-5820K CPU were of the order of 20 minutes per time step (32 time steps were run per load) for Mesh 1, 1.5 hours for Mesh 2 and 12 hours for Mesh 3. In contrast the Author's model run times are of the order of seconds to a minute for a full load step.

4. AUTOMOTIVE EXAMPLE

In this section we discuss a typical automotive application where asymmetric cylindrical gears may be considered as a design option.

Gears in typical automotive applications are mostly subjected to unidirectional loading where the drive flank operates at greater load for longer duration compared to the coast flank. This means that the drive flank dictates the torque capacity of the gears. Asymmetric gears can be designed to increase the performance of the drive flank at the expense of the coast. This can increase the overall load capacity of the gear. Due to this reason there has been increased interest in the use of asymmetric gears within the automotive industry.

The geometry parameters used in this study are given in Table 2 and shown in Figure 10. The original, symmetric design is based on real automotive application. The asymmetric design is an optimized asymmetric alternative to the original gear set. The high contract ratio design (HCR) is the author's symmetric gear optimisation of the original gear set.

Table 2 Design study example geometries.

	Original		Asymmetric		HCR	
	Pinion	Wheel	Pinion	Wheel	Pinion	Wheel
Gear Ratio	2.45		2.53		2.45	
Effective Face Width (mm)	17.5					
Normal Module (mm)	2.2		2.5		2.21	
Helix Angle (°)	23		27.83		23.4	
Centre Distance (mm)	83					
Axial Contact Ratio	0.99		1.0628		1.003	
	Drive	Coast	Drive	Coast	Drive	Coast
Pressure Angle (°)	20		32	16	19	
Transverse Contact Ratio	1.7614		1.08	1.42	1.9943	

Even though, it is possible to design asymmetric gears with high contact ratio the option to do so was limited by the constraints for the example investigated here. One constraint was the both flanks have the same tip form diameter. This interacts with the constraint of maintaining sufficient start of active profile (SAP) to form diameter clearance. Sufficient tip thickness was also maintained for all designs.

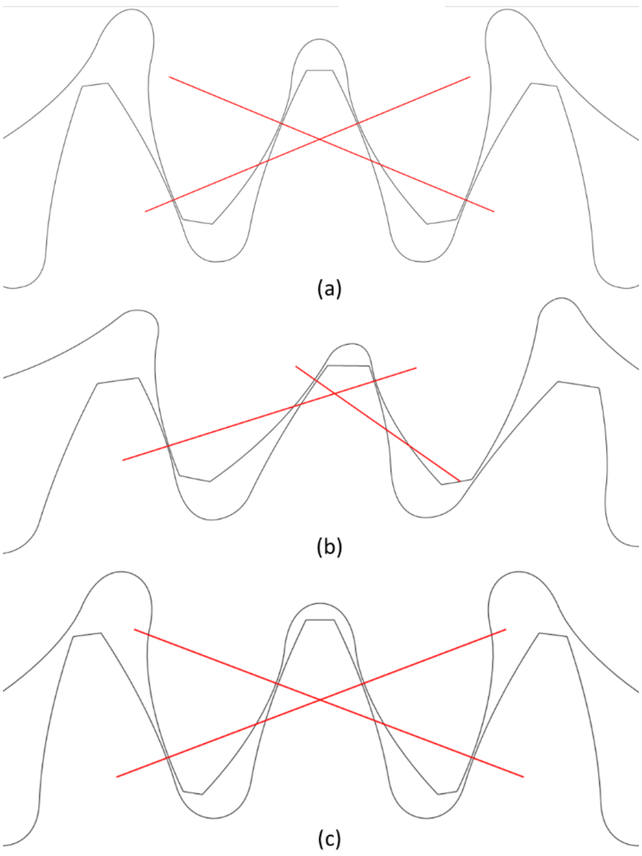


Fig. 10 Comparison of tooth shapes (a) Original, (b) Asymmetric and (c) HCR

These designs were evaluated for peak-to-peak transmission error, contact stress and root stresses using the LTCA methodology described and validated earlier in this paper.

Figure 11 shows calculated peak-to-peak transmission error for the designs detailed in Table 2. The asymmetric gear has substantially reduced transverse contact ratio, and this has an adverse impact on the transmission error. As it can be seen, peak-to-peak transmission error was significantly higher on the drive flank. In the coast flank the asymmetric gear was found to provide a lower peak-to-peak transmission error compared to the Original. The best performing design on the drive flank was the HCR, although it might be possible to achieve improved peak-to-peak transmission error behaviour for asymmetric gears in certain cases as shown by Kapelevich[(2)].

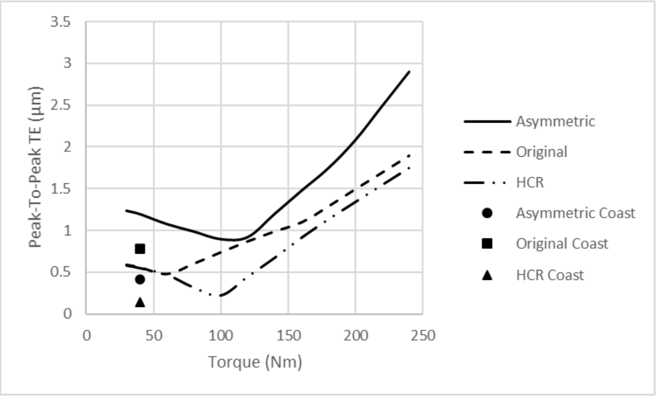


Fig. 11 Comparison of Peak-to-Peak Transmission Error (µm) against load

Figure 12 show the comparison of maximum contact stress for the three designs. For the asymmetric design the maximum contact pressure was reduced compared to the Original. This reduction is much more significant between 50 to 150 Nm than at the higher loads. However the HCR gear resulted in lower contact stresses than the asymmetric gear at all loads. It should be noted that all of these designs have some level of tip contact present.

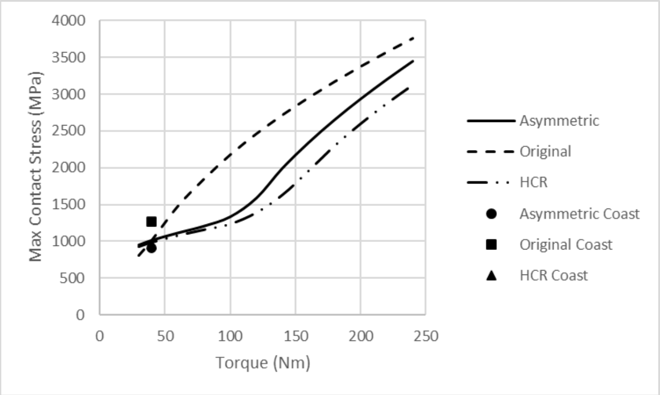


Fig. 12 Comparison of Maximum Contact Stress (MPa) against load

Figure 13 show the comparison of maximum principal root stress, in tension, for the pinon. Using the asymmetric design, maximum tensile stress at the pinion root is reduced by approximately 10 percent in the operating range as compared to the Original design. However, it was found that the HCR gear resulted in similar root stresses to the asymmetric gear.

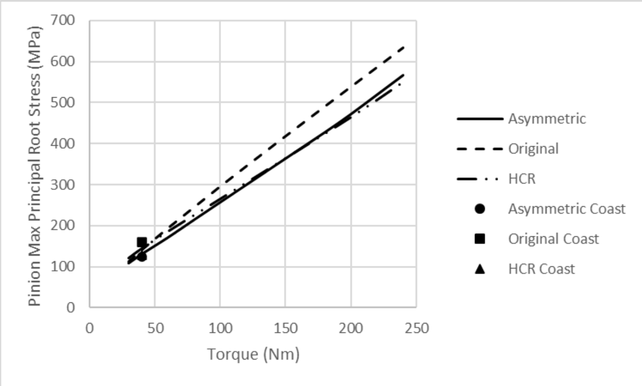


Fig. 13 Comparison of Pinion Max Principal Root Stress (MPa) against load

Figure 14 show the comparison of maximum principal root stress, in tension, for the wheel. The wheel root stresses did not improve for the asymmetric design as compared to the Original whereas they could be improved using a HCR design.

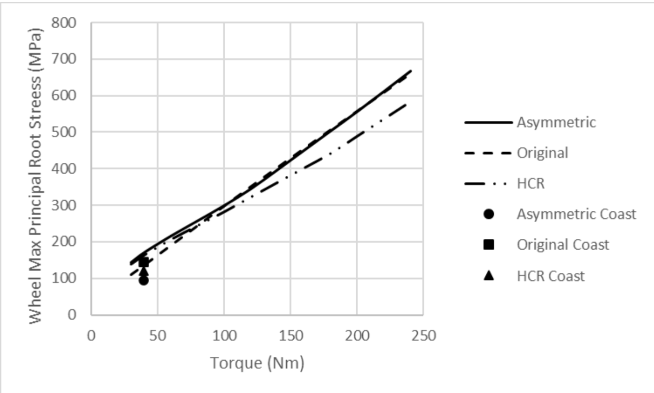


Fig. 14 Comparison of Wheel Max Principal Root Stress (MPa) against load

Asymmetric gear design optimisation for the given example was not very successful, although it reduced pinion root stress and contact stresses when compared to the Original design. For the given constraints it was possible to design a symmetric gear with HCR which was better than the asymmetric gear in every aspect. This indicates that although there are stated potential improvements which can be achieved with asymmetric designs, improvements are not guaranteed. A tool such as that developed by the author's is required to enable engineers to accurately and efficiently compare the advantages and disadvantages of multiple asymmetric designs between themselves and symmetric alternatives.

The designs discussed here were compared based only on transmission error, root bending stress and contact stress. Asymmetric gears might have further advantage if efficiency, scuffing and micropitting are considered. This could result due to improved radius of curvature and specific sliding due to higher pressure angle. In design settings where such criteria are important further analysis is required. However, the efficiency effects must be investigated at the system level as increasing the pressure angle increases the bearing loading. In addition, it is important to understand the cost repercussions of manufacturing and quality control of asymmetric gears compared to symmetric variants.

5. CONCLUSION

Asymmetric gears have been shown in the literature to offer significant operating advantages over symmetric gears in many applications. Increased interest is being seen in the application of asymmetric gears in the automotive industry. An efficient, validated, Loaded Tooth Contact Analysis method has been presented for the assessment of symmetric and asymmetric gear load distribution, transmission error, contact and root stresses. An automotive example was presented showing that potential benefits of asymmetric gears are not necessarily achieved when compared to optimized symmetric gear designs. This highlights the benefits of a tool such as the one presented in enabling the engineer to accurately and efficiently assess multiple gear design options, both symmetric and asymmetric.

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