Tooth Contact Analysis—Off Line of Action Contact and Polymer Gears

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A loaded tooth contact analysis (TCA) model combining an FE representation of bending and base rotation stiffness of teeth with a Hertzian contact formalism for contact stiffness is presented and applied to polymer gears. Comparison with full 3-D FE contact analysis is made. The aim of the study was to apply such a specialized tooth contact analysis method, well-used within the steel gear community, to a polymer gear application to assess what modifications need be made to these models for them to be applicable to polymer gears. It is shown that it is important to include the phenomenon of extended contact at the tips of the gear teeth in such polymer gear tooth contact models for correlation with FE analysis. It was further shown—for the example considered—that the standard assumption made by such models, i.e.—that the points of contact do not move from their theoretical involute contact positions—should also be relaxed in order to capture accurately the shape of the transmission error curve. This may also be the case in certain situations for steel gears. Both effects were implemented in the author’s models and with their inclusion correlation with the 3-D FE contact analysis is good.

Introduction

Gear-loaded tooth contact analysis is an important tool for the design and analysis of gear performance within transmission and driveline systems. Methods for the calculation of tooth contact conditions, with a particular focus on metal gears, have been discussed in the literature for many years. A number of commercial tools are available that perform such calculations (Ref. 1). Such specialized tools are used extensively within the industry for steel gears due to their fast set-up and analysis times. While similarities between tools are significant, they differ in implementation and significant differences in results can be found. The most significant difference between methods is in the representation of gear tooth and blank stiffness used. Methods using a combination of finite element models to capture the bending and base rotation stiffness, and Hertzian formalisms to capture the local contact deflections, are considered among the state of the art.

There are significantly fewer studies in the literature on tooth contact analysis models for polymer gears than there are for steel. Further, the use of such models in the industry is much lower due to questions of validity. Due to their inherent non-linearities, low modulus of elasticity, and significant temperature dependence of material properties, polymer gear tooth contact conditions are significantly more complex than those for steel gears. This study aims to apply an existing, specialized gear TCA model to polymer gear tooth contact and to present modifications that take significant steps towards a model that can be efficiently used as a design-and-analysis tool within the industry.

Performing loaded TCA in a general FE package requires a very fine mesh in order to accurately capture the Hertzian deflections local to the contact, and are therefore very time-consuming to set up and run. As a result, such an approach is rarely used in industry as a design-and-analysis tool. However, it can be considered a benchmark analysis, providing a means of validation of the assumptions made within specialized gear tooth contact analysis models.

In this paper validation results are presented between the author’s specialized gear tooth contact model and the results of a full FE TCA using a commercial FE package showing good correlation for TE, root stress and contact stress.

Gear Tooth Contact Analysis and Polymer Gears

Polymer gears have a number of polymer-specific properties, as compared to steel gears, which may contribute significantly to their tooth contact conditions and therefore a number phenomenon which may need to be included in a specialized gear tooth contact analysis model to accurately model their contact behavior. These include:

- **A larger deflection to permissible load ratio.** Due to their lower modulus of elasticity, deflections are high and the effect of extended tip contact outside the theoretical path of action must be included for polymer gears. As a result, load sharing and operating contact ratio are significantly different from steel gears.
- **Material non-linearity.** The majority of polymer gears are manufactured using thermoplastic materials. Thermoplastics have, for example, different modulus of elasticity under tension and compression, and in a fully detailed analysis their visco-elastic behavior would need to be considered.
- **Temperature dependence of material properties.** Polymers have material properties, such as modulus of elasticity and friction coefficients, which vary significantly with temperature.
- **Humidity dependence of material properties.** As well as temperature dependence, some polymers have significant humidity-dependant material properties.
- **Temperature dependence of geometry.** Polymers have relatively large coefficients of thermal expansion. As such their geometrical dimensions change in a non-negligible way with temperature.
- **Friction.** Friction at the contact is often neglected in the tooth contact analysis of steel gears. However, particularly in the case of dry-running polymer gears, coefficient of friction at the tooth contact is high and can contribute significantly to the tooth contact conditions.
- **Wear.** Certain polymer gears can wear significantly; as they wear, their surface geometry changes and, as a result, their
The focus of this paper is the first item of the polymer gear contact analysis. The aim of the study is rather a first step in the process—to begin with a class of specialized tooth contact analysis methods that are well-used within the steel gear community and to apply them to a polymer gear application to assess whether modifications need be made to these models to make them applicable to polymer gears. The focus of this paper is the first item of this study to cover all these contributing factors to polymer gear tooth contact analysis. The aim of the study is rather a first step in the process—to begin with a class of specialized tooth contact analysis methods that are well-used within the steel gear community and to apply them to a polymer gear application to assess whether modifications need be made to these models to make them applicable to polymer gears.

It is not the intention of this study to cover all these contributing factors to polymer gear tooth contact analysis. The aim of the study is rather a first step in the process—to begin with a class of specialized tooth contact analysis methods that are well-used within the steel gear community and to apply them to a polymer gear application to assess whether modifications need be made to these models to make them applicable to polymer gears. The focus of this paper is the first item of this study to cover all these contributing factors to polymer gear tooth contact analysis. The aim of the study is rather a first step in the process—to begin with a class of specialized tooth contact analysis methods that are well-used within the steel gear community and to apply them to a polymer gear application to assess whether modifications need be made to these models to make them applicable to polymer gears.

Some important considerations for polymer gears, such as the temperature dependence of the material, are largely ignored within this study, although an appropriate modulus of elasticity corresponding to the operating conditions is used. It is not expected therefore that the TCA results obtained reflect the full detail of the real tooth contact conditions of the polymer gears under consideration.

Although less extensive than the literature regarding steel gear tooth contact analysis, there still exist many notable contributions regarding polymer gear TCA within the literature. It is not the intention here to present a full literature review; however, we shall introduce some relevant papers of interest—particularly those with regards to FE and specialized tooth contact models as validated against full FE.

Polymers have lower accuracy and therefore larger manufacturing errors—such as pitch errors. Any specialized gear tooth contact program should provide the option to include these errors as input to the analysis.

Extended tip contact in the context of polymer gears has been investigated by Eritenel et al (Ref. 7). The focus was the proposed problem of unloaded flank tooth contact due to excessive bending. Eritenel et al used the TCA program CALYX to show that this does not occur in their examples, and the deflection on the backside increases with increasing load due to the relative contributions from Hertzian and bending deflections.

Letzelter et al (Refs. 8–9) developed a model using a specialized, gear-loaded tooth contact analysis that includes the visco-elastic properties of the materials via the use of the generalized Kelvin model. Results of the model are presented for Polyamide 6/6. Validation via prediction and measured TE is presented, showing relatively good agreement. No thermal or mechanical coupling is considered in the model. Cathelin et al (Ref. 10) develop a related method for computing the loaded mechanical behavior of fiber-reinforced polymer gears. The influence of the fiber orientation is considered via the FE models used in the method for the bending and base rotation influence coefficients. Results are presented for Polyamide 6 with 30% glass-fiber reinforcement.

Raghuraman (Ref. 11) developed a pre-processor to the tooth contact analysis program LDP—“Plastic Gear Program (PGP)”—to consider the effect of temperature/humidity and tolerance by altering the microgeometric and macro-geometric parameters before a loaded TCA is performed in LDP.

Methodology
Specialized loaded tooth contact analysis model. The specialized loaded tooth contact analysis model used in this study is presented in detail (Ref. 1). As was assumed for steel gears (Ref. 1), dynamic effects are not considered in this study.

A common assumption made (in Ref. 1)—and usually in the type of specialized gear tooth contact analyses discussed here—is that deflections and microgeometry are sufficiently small that the contact points and normals do not move from their theoretical no-load locations.
This assumption is implicitly not made in the FE analyses presented, where surface-to-surface contact elements are used and the region of contact calculated during the analysis. This assumption is also relaxed during this study. A modification to the model presented (Ref. 1) is made to take into account the change in contact point up the gear tooth profile resulting from the deviation from involute geometry due to applied flank modifications. It is shown that this modification is required for good correlation of our specialized models with our FE contact analysis results.

A brief description of the method is presented here for brevity; further details can be found in (Ref. 1).

- Inputs to the calculation include torque, gear macro and micro geometry (flank modifications) and misalignment at the gear mesh. The analysis is quasi-static. At each time step, unloaded, potential contact lines are first calculated from the gear macro geometries, relative locations and rotations. Potential contact lines are divided into strips and contact points expressed in a 2-D coordinate system as distance along face width and roll angle.
- Points that could come into contact at the tips of one of the gears due to tooth deflections are included in the discrete set of potential contact points. The gap between the contacting flanks at these points is calculated according to (Ref. 5).
- During the process of this study, modifications to the calculation presented (Ref. 1) were made to account for the change in potential contact points in terms of their positions up the profile due to micro geometry modifications. Figure 1, for example, shows the points that would come into contact as the driving gear, top of the picture, rotates anti-clockwise due to bending, into contact with the tip of the driven gear. Without tip relief on the driven gear the contact points indicated by triangles will contact first, while with tip relief the contact points indicated by squares will contact first. The gap between flanks due to the micro- and macro-geometry at the points indicated by the triangles and squares is different, and so the calculated transmission error is different depending on which pair is taken as the points of potential contact. At each mesh position the calculation was modified to search the flank in the profile direction around the nominal contact points (without microgeometry) to find those with the minimum gap between flanks when the designed microgeometry is included; these points were used as the potential contact points in the rest of the calculation, instead of the nominal, no microgeometry, points.
- Compatibility and force equilibrium conditions relating the discretized contact points are formulated and solved.
- Results include the load distribution across the contact lines, the elastic deformations at each contact point pair, and transmission error. Load distribution results are further used to calculate contact stresses on the gear flanks.

In the class of models considered here, the elastic deformations are separated into two parts. The bending stiffness and base rotation of the teeth are included via an FE model of the gear. The Hertzian contact stiffness of each strip is considered separately via a Hertzian line contact formalism.

In our implementation the FE mesh used for bending and base rotation stiffness for each gear is generated from the exact gear macrogeometry using the same code that generates the full FE tooth contact analysis meshes discussed later. Via this FE representation, the compliance due to loads on adjacent teeth is naturally considered. The stiffness with respect to the regular FE grid on the gear flanks is calculated via Guyan reduction of the FE stiffness of the full gear. The stiffness with respect to potential contact points, which will not coincide with the nodes of the regular grid, is interpolated using the shape functions of the FE elements. Root stress influence coefficients are also calculated from the FE model for unit loads at each flank location.

It is only required to perform these steps once for each gear macrogeometry. It is reasonably assumed that the microgeometry modifications do not significantly affect the bending stiffness of the FE model. Therefore microgeometry and misalignments can be changed, and the TCA rerun without having to recalculate the bending stiffness.

The local contact between potential contact points is considered as a line contact between cylinders. The compression of each tooth between the point of load and the center line is also included, as this is removed from the stiffness represented by the FE model. In our implementation, the approach of Weber (Ref. 12) was chosen.

The load distribution calculated during the analysis together with the root stress influence coefficients from the FE model are used to calculate the root stress in the gears throughout the mesh cycle.

Figure 1  Potential contact points under tooth bending. Triangles — without microgeometry; squares — with microgeometry (parabolic tip relief).
Loaded Tooth Contact Analysis Model in FE

For validation of our model, loaded tooth contact analysis was performed in ANSYS. The method used to set up and run the FE tooth contact analyses in ANSYS was presented in detail in (Ref. 1).

The geometry was specified in SMT’s MASTA software (Ref.13). An algorithm was written to define the FE mesh node locations in an ANSYS Parametric Design Language (APDL) script directly from the geometry, thus avoiding issues that can arise if the geometry is constructed via a CAD model. Microgeometry flank modifications were included. The algorithm generates a 3-D mesh for a single tooth section that is then duplicated, rotated and merged to generate a mesh for multiple teeth. A sufficient number of teeth are included in the FE model to capture the effect of adjacent teeth on those teeth in contact. The rest of the gear blank is generated as a cylinder from bore-to-root diameter. Misalignment is included by modifying the node positions. Gears were positioned in mesh and backlash was removed by a further rotation of the pinion, as it is not included in the specialized gear tooth contact calculations used.

The solid mesh consists of quadratic SOLID186 elements, as required for accurate calculation of the contact pressures between teeth. Linear elements were used (Ref. 1), as the focus there was TE—not stress. Surface-to-surface contacts were defined between the potentially contacting teeth. The Lagrange method was used to maintain the contact constraints purely via Lagrange multipliers, as such no penetration between contacts was allowed.

Figure 2 shows the boundary conditions applied to the FE model. Zero displacement boundary conditions were applied to all degrees of freedom at the bore of the wheel. The torque was applied at the pinion bore via a pilot node at its center. The pilot node was rigidly connected to the pinion’s bore in all degrees of freedom using rigid node to surface constraints. Zero displacement boundary conditions were applied to all degrees of freedom, except rotation about the pinion axis, at the pilot node.

A static analysis was run at 32 mesh positions and the rotations of the pinion about its axis were written to file. Linear TE was calculated as the pinion rotation multiplied by its base radius. Maximum contact pressures and maximum principal tensile root stresses at each mesh position were also written to file. Geometric non-linearity was included in the analysis. Force convergence was checked.
A mesh refinement study was performed for all results. The critical area for refinement in a gear tooth contact problem is at the tooth contacts themselves. A mesh fine enough to capture the Hertzian contact deformation is required. For the increasing levels of mesh refinement, the mesh was refined in all areas, with more refinement at the contacts. Figure 3 shows examples of the meshes used and results obtained from the FE analysis.

It is informative to note the relative run times for the 3 meshes considered. Using a 64-bit system with Intel Core i7-5820K CPU @ 3.30 GHz and 64 GB of RAM, for 32 mesh positions the analysis for Mesh 1 ran in approximately 17 hours. Mesh 2 ran in approximately 42 hours while Mesh 3 took approximately 230 hours. This indicates why full FE analysis is rarely used as a design-and-analysis tool for gear tooth contact and why specialized gear tooth contact models, which have run times of the order of seconds to minutes, are used extensively in industry.

**Results**

The machine-cut acetal gear set presented (Ref. 14) was chosen as our test case. These are standard gears, cut using a standard basic rack and operating at the standard center distance. The gear geometry and material details are presented in Table 1.

It was assumed that the modulus of elasticity was 1,700 (MPa), a value for Delrin 100 at approximately 60 degrees centigrade. The temperature dependence of the modulus was not considered. This is a crude assumption for polymers, as discussed in the previous section.

Both gears had 50 µm of parabolic tip relief.

Figure 4 shows the transmission error trace for the 10 Nm load case for a number of models. A number of conclusions can be made from these results. One, it is clear that the closest correlation with the ANSYS results is for the author’s model that includes the effect of extended tip contact and the effect of the change in contact position due to microgeometry. Correlation between these models is good in terms of peak-to-peak TE, mean TE and the overall shape of the TE curve, indicating close agreement in all harmonics of TE. The author’s model with extended tip contact, but not includ-

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**Table 1  Machine-cut acetal spur gear pair example (Ref. 14)**

<table>
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<tr>
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<th>Wheel</th>
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<td>30</td>
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<tr>
<td>Normal Module (mm)</td>
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<td>2</td>
</tr>
<tr>
<td>Normal Pressure Angle (degrees)</td>
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<td>20</td>
</tr>
<tr>
<td>Face Width (mm)</td>
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</tr>
<tr>
<td>Cutter Edge Radius (mm)</td>
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<td>Max</td>
</tr>
<tr>
<td>Bore Diameter (mm)</td>
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<td>16</td>
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<td>Tooth Thickness at Reference Diameter (mm)</td>
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</tr>
<tr>
<td>Contact Ratio</td>
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</tr>
<tr>
<td>Poisson Ratio</td>
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</table>
ing the change in contact position due to microgeometry, agrees well in terms of mean and peak-to-peak TE with the ANSYS results. However, the shape of the TE trace is significantly different. The results for the author’s model without extended tip contact, together with the observations above, show that extended tip contact is playing a key role in the peak-to-peak, mean and shape of the TE trace in this case.

Figures 5 and 6 present mean transmission error and peak-to-peak transmission error against torque for a range of loads. As with the results presented above, it was observed at all loads considered that the effect of the change in contact point due to microgeometry on the author’s results for mean and peak-to-peak transmission error is negligible. Results are therefore only presented with this effect included. A number of conclusions can be made from these figures.

One, for mean transmission error there is reasonable correlation between all models. At higher loads the author’s model, including the effect of extended tip contact, deviates slightly from those without the effect of extended tip contact where these tip contact points are taking some load resulting in a stiffer mesh. The author’s results with extended tip contact match those of ANSYS very closely.

For peak-to-peak transmission error, clear differences exist in results between some models. For loads at and below 6 Nm, correlation between results is reasonably good. After 6 Nm the extended tip contact points begin to be loaded and the author’s results with extended tip contact deviate significantly from those without. Again, correlation between the author’s results with extended tip contact and the ANSYS results match very closely.

Figure 7 presents the operating contact ratio as calculated by the author’s models. It is clear that the theoretical contact ratio is exceeded at around 2 Nm; this indicates that at this point the extended tip contact points are coming into contact. At approximately 6 Nm the operating contact ratio increases above 2. Figures 5 and 6 show that it is at 6 Nm that the mean and peak-to-peak transmission error are significantly affected by the extended tip contact. This is expected for spur gears, as the extended tip contact points come into contact earlier; the region of single
tooth contact shrinks as load increases, until the point where the operating contact ratio is pushed over 2 and no region of single tooth contact remains.

Figure 8 presents the results for maximum principal tensile root stress on the pinion; the ANSYS and MASTA results agree well at all loads.

Figure 9 presents the results for maximum contact stress. Contact pressure results for such a general surface-to-surface contact problem in FE are among the most difficult results to obtain accurate values for. The ANSYS results presented are for a quadratic mesh where the maximum contact pressure curve against phase of gear mesh has been smoothed using a Savitzky—Golay filter. Convergence has been checked on the smoothed results. Smoothing of the contact pressure results is required due to numerical errors and inaccuracies in the details of the contact algorithm in ANSYS. The results show good agreement with the author’s models. It is worth noting that the author’s model post-calculates the contact pressure from the calculated load distribution using a Hertzian formalism by assuming each contact point pair as cylinder-on-cylinder contact.

Conclusions

• A specialized gear tooth contact analysis model based on a hybrid FE and Hertzian contact formalism has been presented and applied to an example pair of polymer gears.
• An extensive comparison was shown between the results of this model and a 3-D FE tooth contact analysis using ANSYS showing excellent correlation in transmission error, root stress and contact stress results.
• It was shown that, as expected for polymer gears, the extended off-line of action tooth contact at the gear tips plays a critical role in transmission error. It was further shown that the standard assumption made by such models — that the points of contact do not move from their theoretical involute contact points — also should be relaxed in order to capture accurately the shape of the transmission error curve for the polymer gears under consideration. This effect can also be important for steel gears.
• The presented solution has been implemented in SMT’s MASTA software.
This study provides a step towards developing a TCA tool that can be efficiently and accurately applied to the design and analysis of polymer gears. The author considers the main step remaining in the analysis of the tooth contact conditions of polymer gears is the inclusion of a more detailed representation of the temperature dependence. For a proper treatment of this behavior, a coupled thermal and mechanical model will be required.

References


Dr. Paul Langlois is the CAE products development department manager at SMT. Having worked for SMT for 10 years, he has extensive knowledge of transmission analysis methods and their software implementation. He manages the development of SMT’s software products and is a main contributor to many aspects of SMT’s technical software development. As a member of the BSI MCE/005 committee, Langlois contributes to ISO standards development for gears.